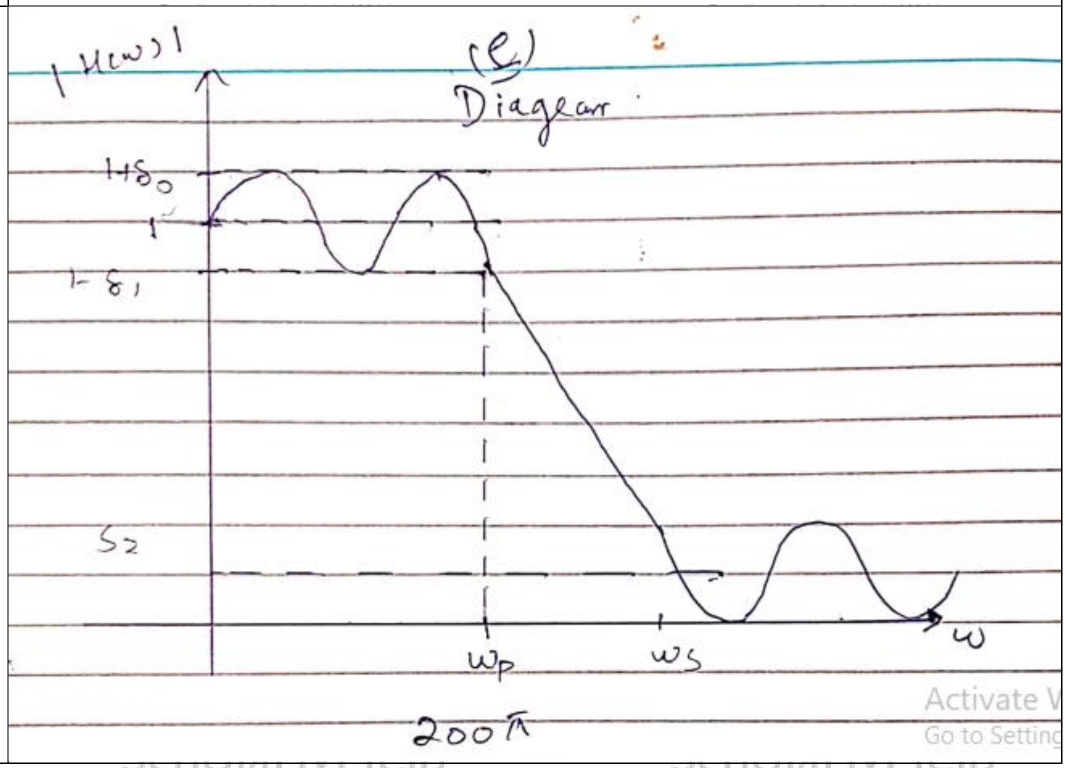
I. AIM:

This lab has two major parts pre-work and lab work, pre-work majorly consists on mathematical work and calculation this parts aims on designing and defining of filters IIR filter and FIR filter. We have defined both types of filter in the table-1 given below. Following the definition of two filters we are required to write the system and difference equation for both the filters. The next steps aim on sketching a signal using the MATLAB software by writing a suitable code for the requirements. Finding the poles and zeros of a notch IIR filter with the specifications given and drawing a diagram for the result is also a requirement for this part.

The second major part Lab work with two subparts Part and Part B focuses on designing of different kinds of filter like Butterworth IIR filter, Chebyshev Type I IIR, Equiripple FIR filter with Low pass filter design and same filters with band-pass filter design. In part B we are required to illustrate and explain any kind of difficulty and problem we have faced in the Part A. In short this lab practical aims on designing and defining of different kinds of filters.

II. Table 1: Basic concepts of discrete-time filters and design of notch filter.

a. What is an IIR filter? What is an FIR filter?	IIR filter requires the current output data and past output data as well. The performance of IIR filter is considered better, as it uses a feedback mechanism which results in an infinite impulse response. Comparing to the IIR filter the response of FIR filter is finite, as in a finite period of time it settles to zero.
b. Give an example of an IIR filter expressed in the form of a difference equation and in the form of a system function.	System Function Form $H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$ In Difference equation form: $y(n) = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$
c. Give an example of an FIR filter expressed in the form of a difference equation and in the form of a system function.	System equation form: $H(z) = \sum_{k=0}^{M} b_k z^{-k}$ $= b_0 \prod_{k=1}^{M} (1 - c_k z^{-1})$ Difference equation form. $y[n] = \sum_{k=0}^{M} b_k x[n-k]$

d. What is the discrete-time pass-band edge (angular) frequency ω_p for the discrete-time filter?	**Discrete frequency = 0.4pie rad/sec**
e. Sketch the magnitude response of a discrete-time filter ... (Pass-band = [0, 200] Hz).	

f. Sketch the magnitude response of another discrete-time filter ... (Pass-band = [100, 200] Hz).	(f) Data: Given: Sampling frequency $F_s = 1000$ Hz Pass-band = [100, 200] Hz Stop-band 1 = [0, 50] Hz Stop-band 2 = [250, $F_s/2$] Hz Pass-band ripple = 3dB Stop-band attenuation = 40 dB Signal $20\log_{10}\lvert H(j\Omega)\rvert = 20\log_{10}\lvert H(j2\pi F)\rvert$ Solution: Frequency range $[0, F_s/2]$ Hz $\omega = \frac{2\pi f}{F_s}$ Pass Band Frequency = $\frac{2\pi(2000)}{1000} = 0.4\pi$ rad

Stop band frequency $= \frac{2\pi(250)}{1000}$

$= 0.5\pi$ rad.

So, it is within the interval $1 \pm \delta$

$20\log(1+\delta) = 3$

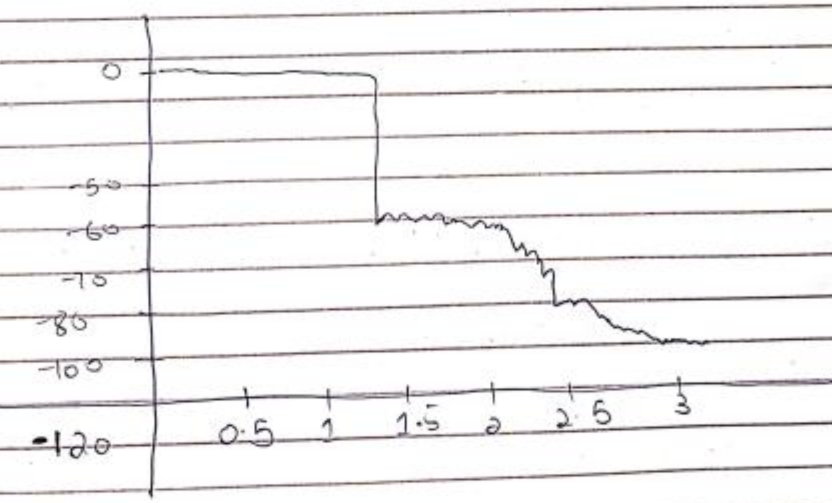
$\delta = e^{0.15} - 1 = 0.16183$

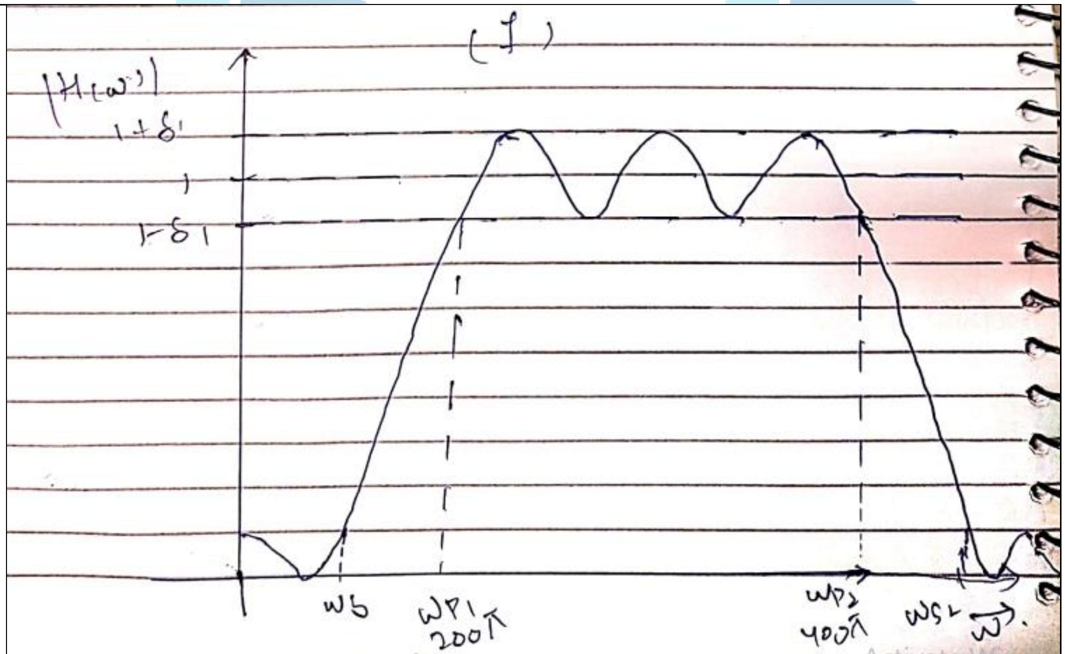
" So, the passband's frequency response is within the interval. "

Result:

$0.8382 \le H(\omega) < 1.6183$

★ Stop band maximum values $= |H(\omega)| < 10^{\frac{-40}{20}} = 0.01$



	
g. Find the coordinates (in the polar form) of the poles and zeros of an IIR notch filter ...	**R = 0.938** (iv) putting the values of $R = 1 - 2\pi \frac{\Delta f}{\frac{f_s}{2}}$ $\Rightarrow \boxed{0.938}$ → Answer for $\boxed{R}$

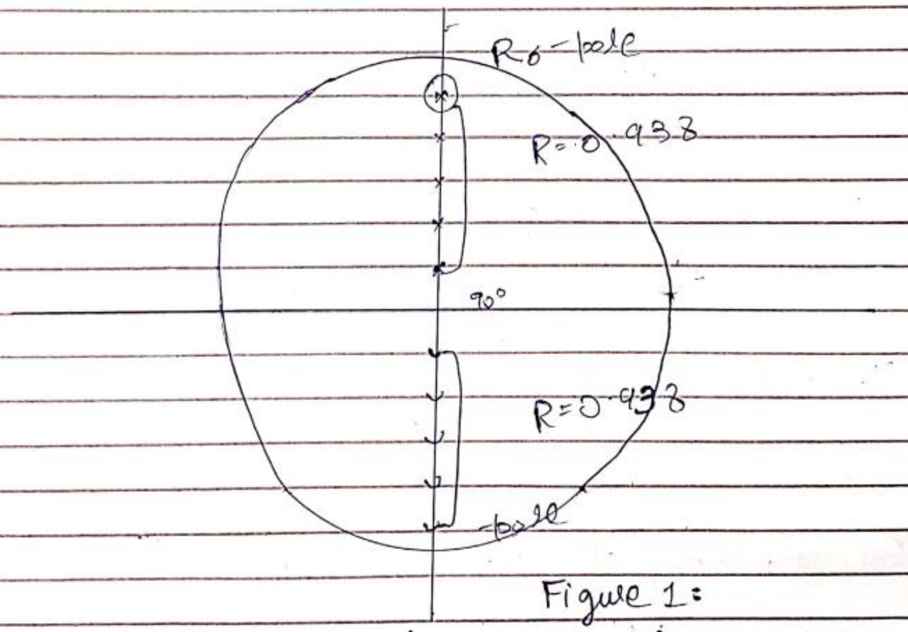
<table>
<tr>
<td>h. Sketch the pole-zero diagram of the notch filter, indicating the locations of the poles and zeros.</td>
<td>→ Diagram:


Figure 1:
The above ~~filter is~~ figure is filter pole zero</td>
</tr>
</table>

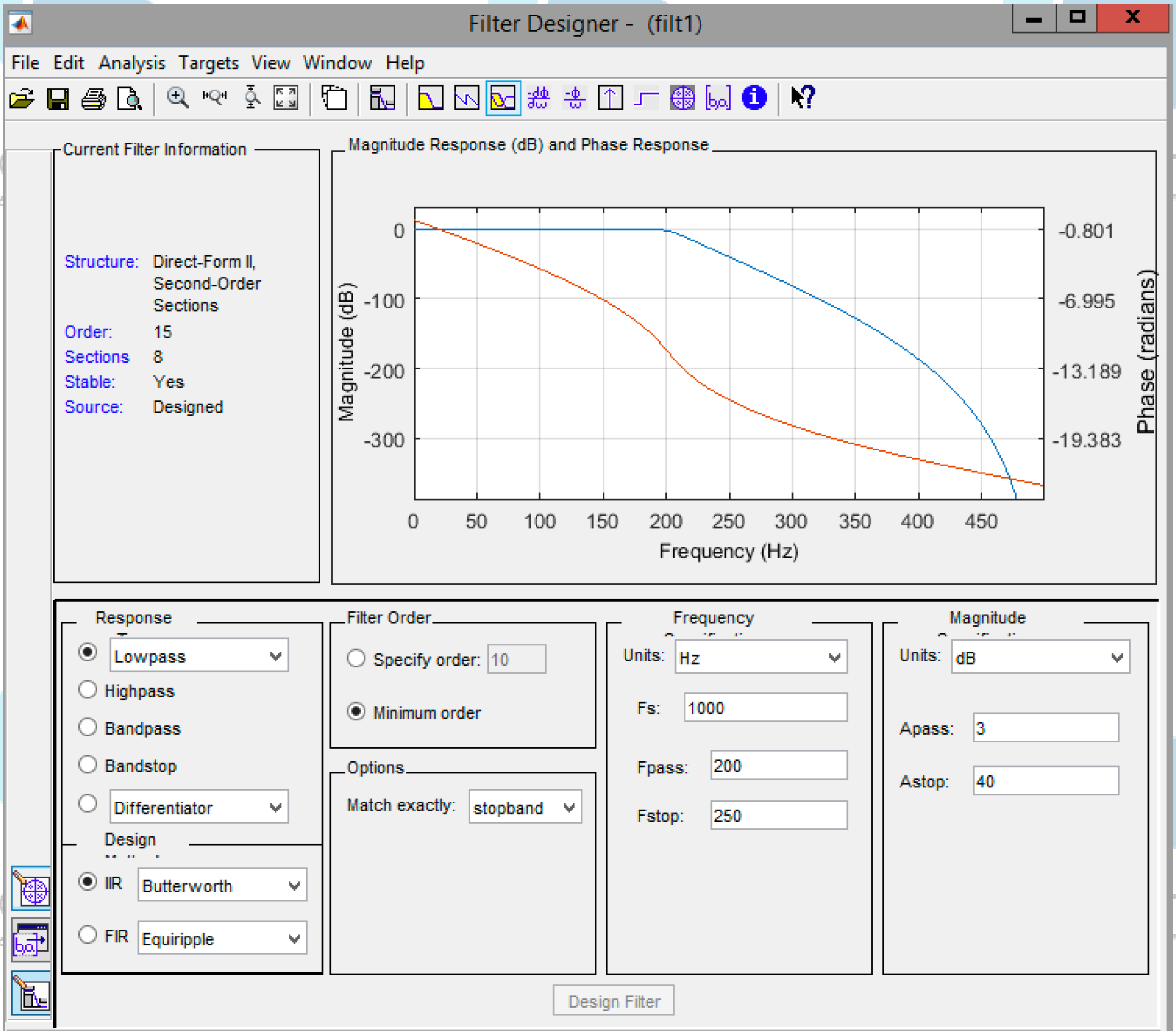
III. LAB WORK PART A:

1. Low-pass filter (LPF) design

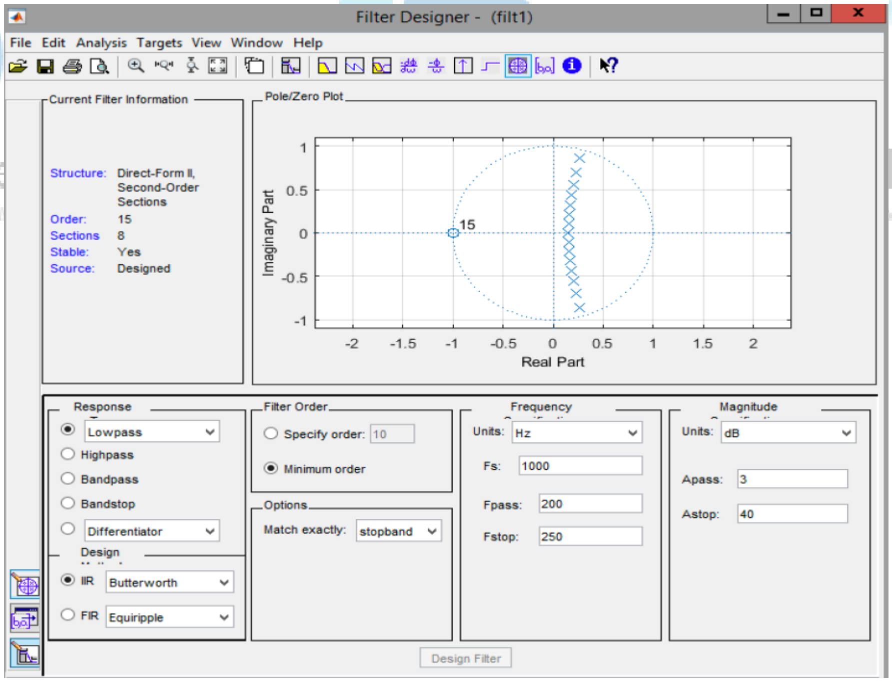
a) Design a Butterworth IIR filter with the following specifications.

- Sampling frequency: 1000 Hz.
- Pass-band edge frequency: 200 Hz.
- Pass-band ripple: 3 dB.
- Stop-band edge frequency: 250 Hz.
- Stop-band attenuation: 40 dB.
- Minimum order design.

Magnitude response and phase response of the filter



Pole/zero diagram of the filter

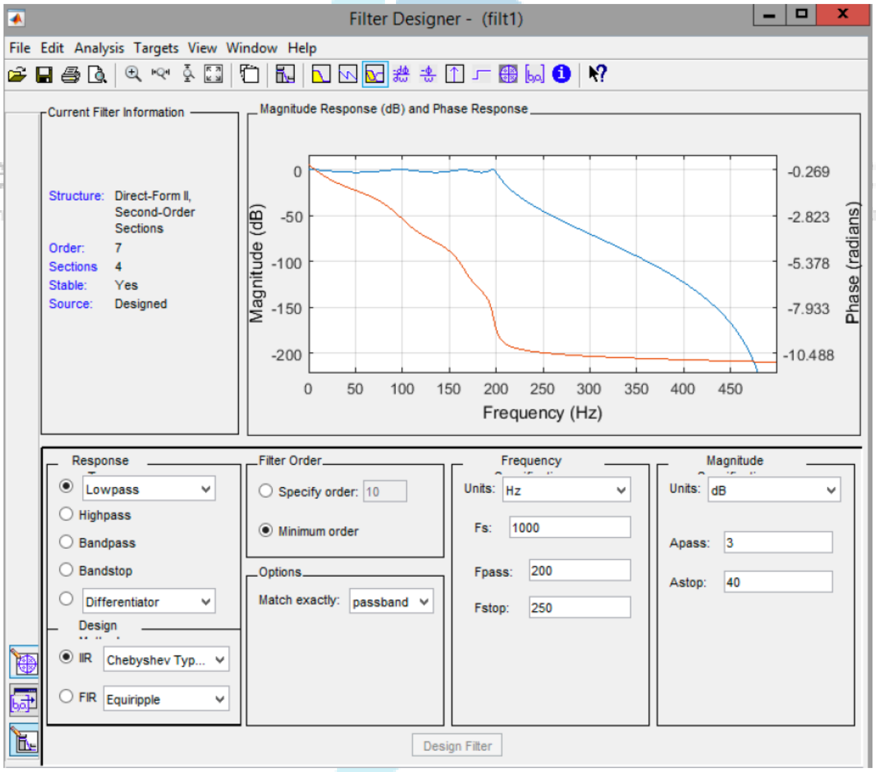


Order of filter

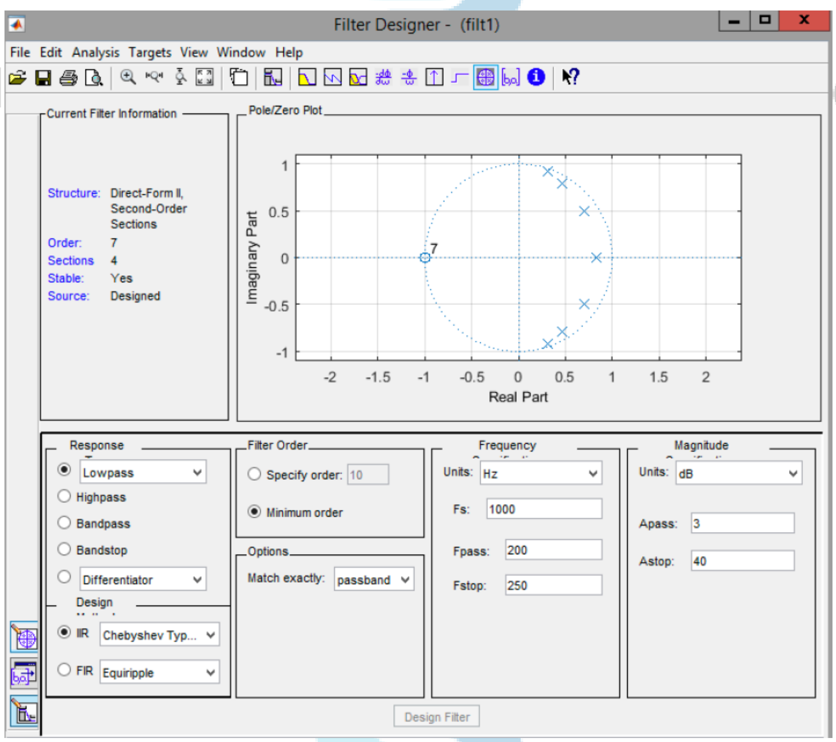
Order of the filter is 15 as the number off zeros is not equal to the number of poles, so we will take the larger number as the order of the filter which in number of poles and, the number of poles are 15.

b) Design a Chebyshev Type I IIR filter with the same specifications as in Task 1a.

Magnitude response and phase response of the filter



Pole/zero diagram of the filter

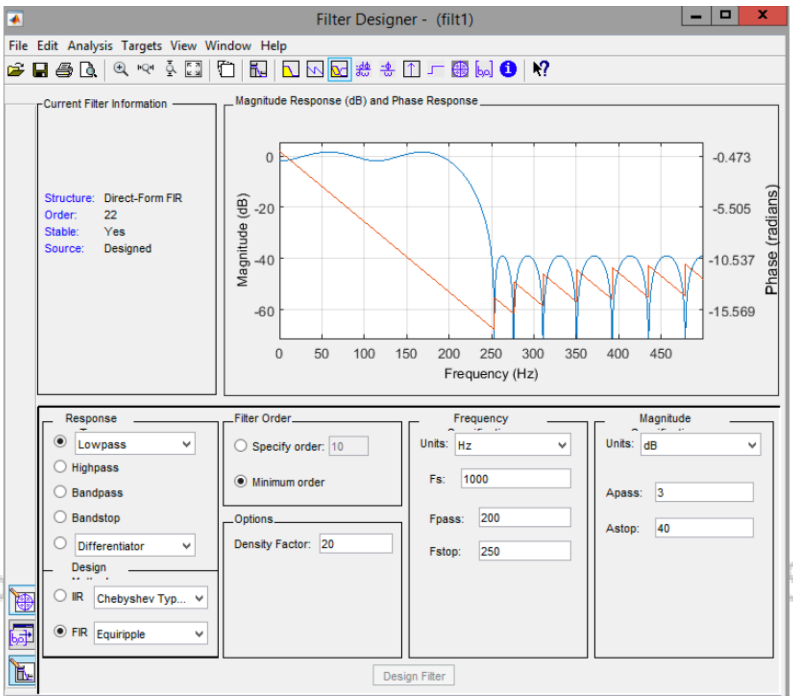


order of the filter

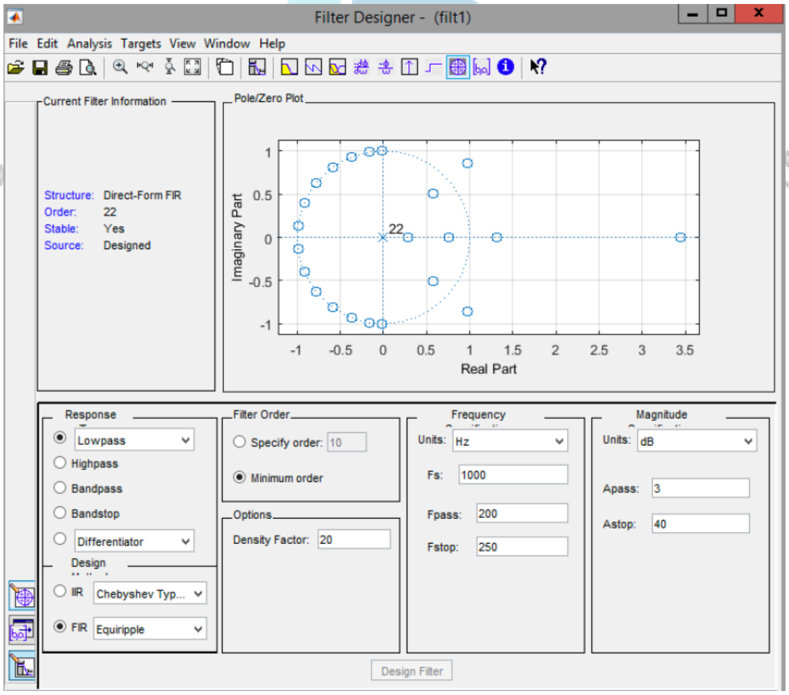
Order of the filter is seven, as the number off zeros is not equal to the number of poles, so we will take the larger number as the order of the filter which in number of poles and, the number of poles are 7.

c) Design an equiripple FIR filter with the same specifications as in Task 1a.

Magnitude response and phase response of the filter



Pole/zero diagram of the filter



order of the filter

Order of the filter is 22 as the number off zeros is not equal to the number of poles, so we will take the larger number as the order of the filter which in number of zeros and, the number of zeros are 22.

d) Compare the magnitude response, phase response and order of three low-pass filters you have designed. This task can be completed when you write your report after the lab class.

The requirements for this task (Task 1d) are listed below.
i. Describe the difference of three filters in terms of magnitude response;

ANSWER: In terms of magnitude response the Butterworth IIR filter starts from zero with a linear line parallel to the x-axis till it reaches a frequency of 200Hz, and then it gradually decreases and reaches to zero at 425Hz. In case of Chebyshev Type I IIR filter the magnitude response is almost same as in the first case but the only difference is instead of a pure linear line from zero to 200 Hz there is some noise in the graph as you can see from the plot, after 200 Hz the slope gradually decreases and the magnitude value is zero at 425 Hz.

The magnitude plot of the third filter is completely different from the above mentioned filter as it is a equiripple FIR filter. For the first time its magnitude reaches to zero at 250 Hz, and then there is a repeated half cos wave from the 250 Hz till 425Hz.

ii. Describe the difference of three filters in terms of phase response;

ANSWER: The values of the phase of the three filters are:

Butterworth filter: -24.577 to -0.701

Chebyshev Type I IIR filter: -10.988 to -0.269

Equiripple FIR filter: -18.069 to -0.473

The phase of the above two filters gradually increases with negative sign. While in third case we have a repeated value for the frequency range of 310 to 425 Hz.

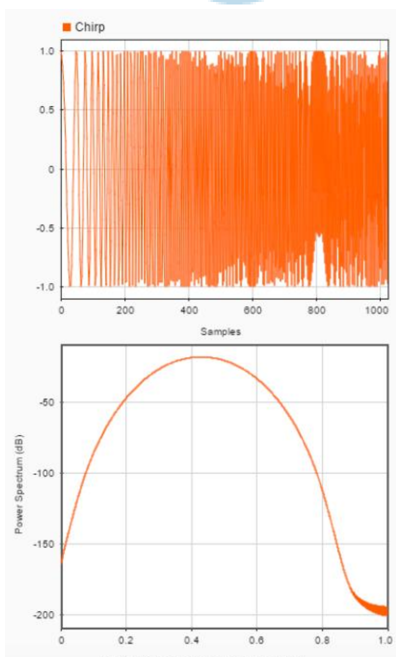
iii. Describe the difference of three filters in terms of filter order.

ANSWER: The order of the three filters as per the SP tool in MATLAB and by noticing and visualizing the number of zeros and poles is 15, 7 and 22 respectively according to the above mentioned sequence in question 2.

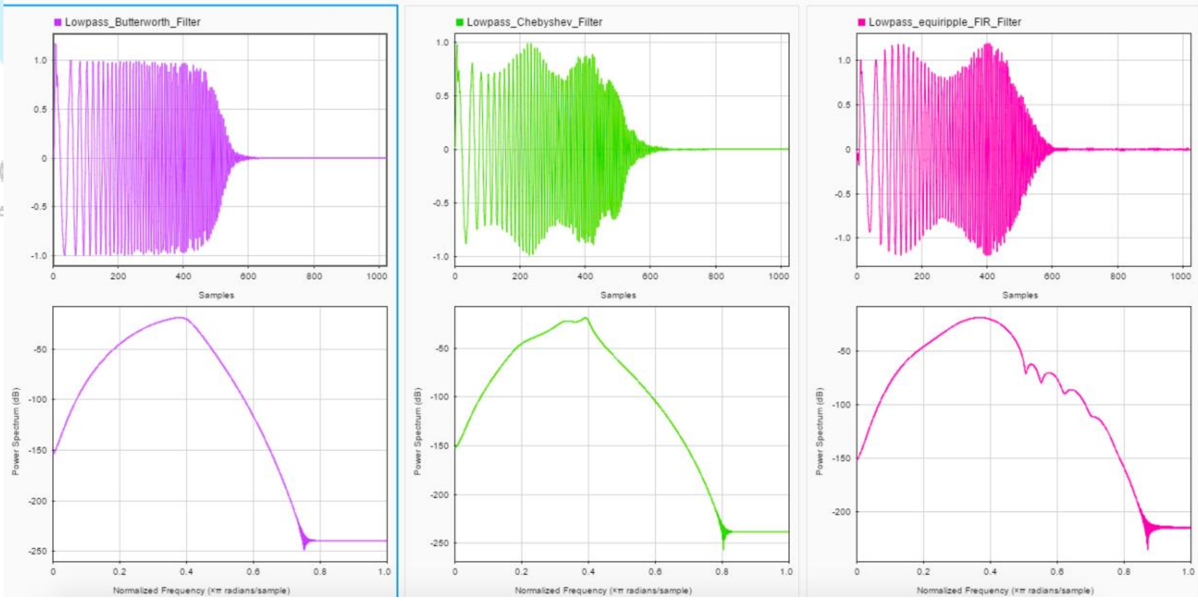
Filter the signal 'Chirp.mat' with each of the above low-pass filters. The sampling frequency for the 'Chirp' signal is 1000 Hz

The requirements for this task (Task 1e) are listed below.

i. Save the figures showing the input signal in the time domain and in the frequency domain and then include the figures in your lab report.



ii. Save the figures showing three output signals in the time domain and in the frequency domain and then include the figures in your lab report.



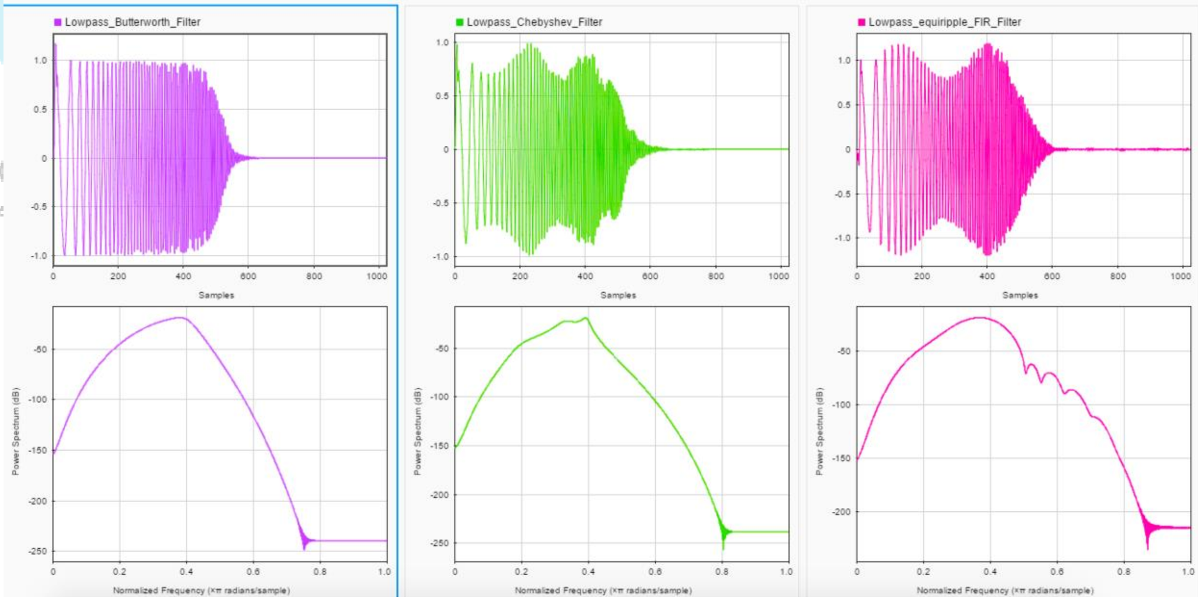
iii. Describe the difference between the input signal and the output signal in the time domain and in the frequency domain for each filter.

ANSWER: In the time domain as for the low pass Butterworth filter the y-axis value maintains the +1 and -1 value and then it approximately approaches to zero like the other two filters. The chebyshev and equiripple low pass filter time domain plot is approximately same. The only difference is in equiripple the y-axis value increases and then decreases and it shows this behavior two time while in chebyshev this repetition was done three times.

In the frequency domain all the three plots are similar, but the only difference is in equiripple plot, in which the y-axis values increases and decreases for the frequency range of 0.5 to 0.7 (*pie radians/sample).

iv. Describe the difference of the filtering results using three filters.

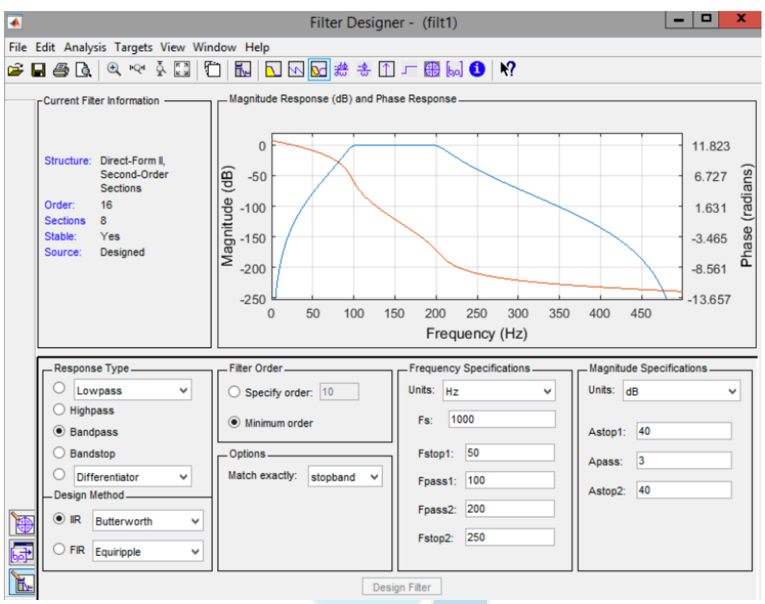
ANSWER:



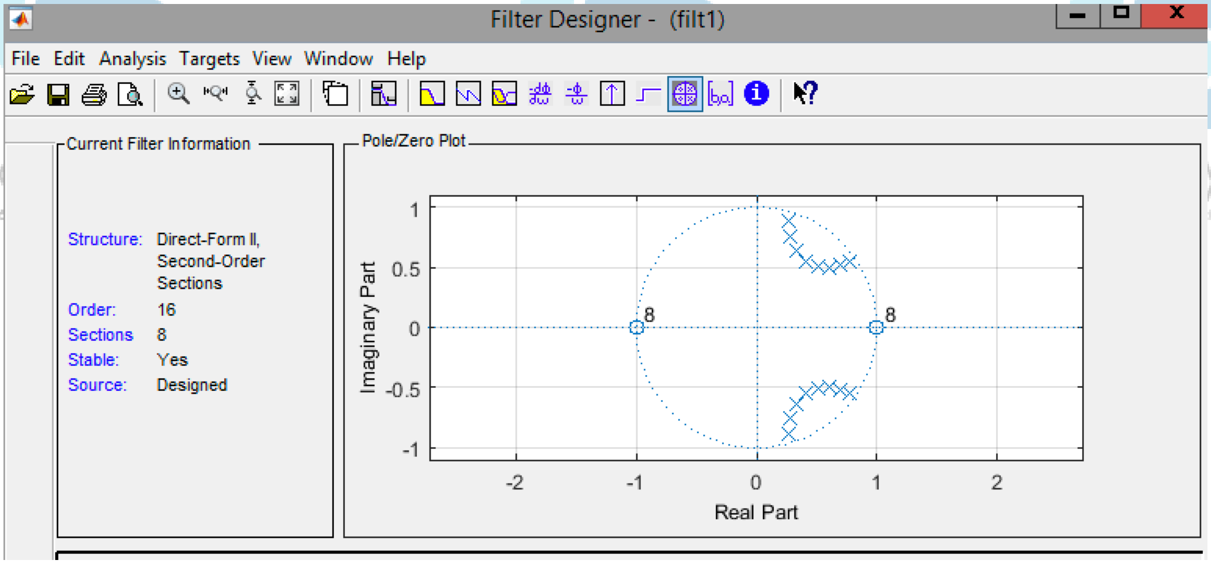
2. Band-pass filter (BPF) design

a) Design a Butterworth IIR filter with the following specifications.
   - Sampling frequency: 1000 Hz.
   - Pass-band edge frequencies: 100 Hz and 200 Hz.
   - Pass-band ripple: 3 dB.
   - Stop-band edge frequencies: 50 Hz and 250 Hz.
   - Stop-band attenuation: 40 dB.
   - Minimum order design.

Magnitude response and phase response of the filter



Pole/zero diagram of the filter

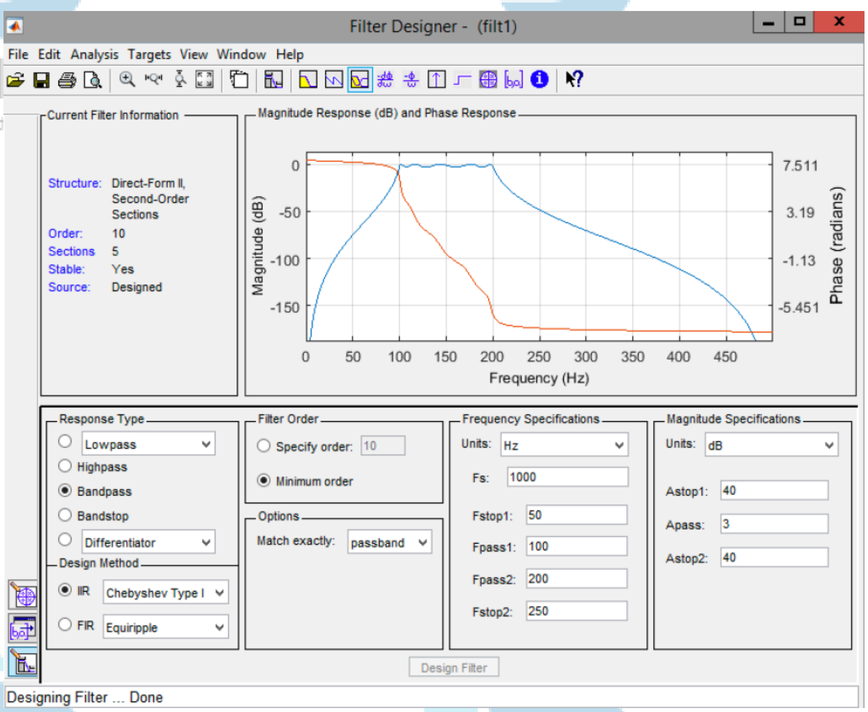


order of the filter

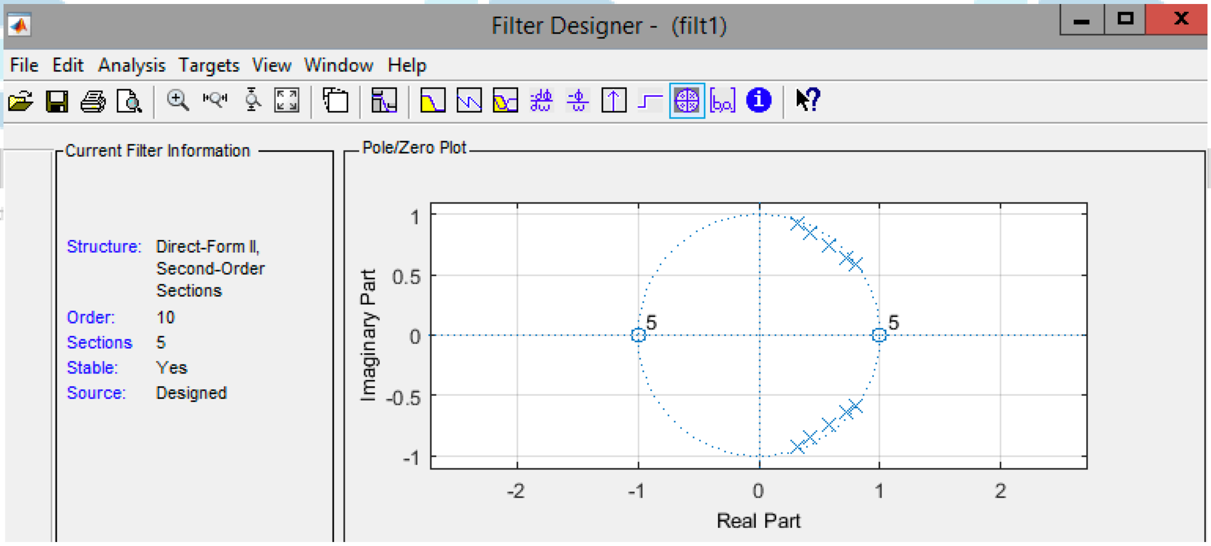
18 is the order of the filter

b) Design a Chebyshev Type I IIR filter with the same specifications as in Task 2a.

Magnitude response and phase response of the filter



Pole/zero diagram of the filter

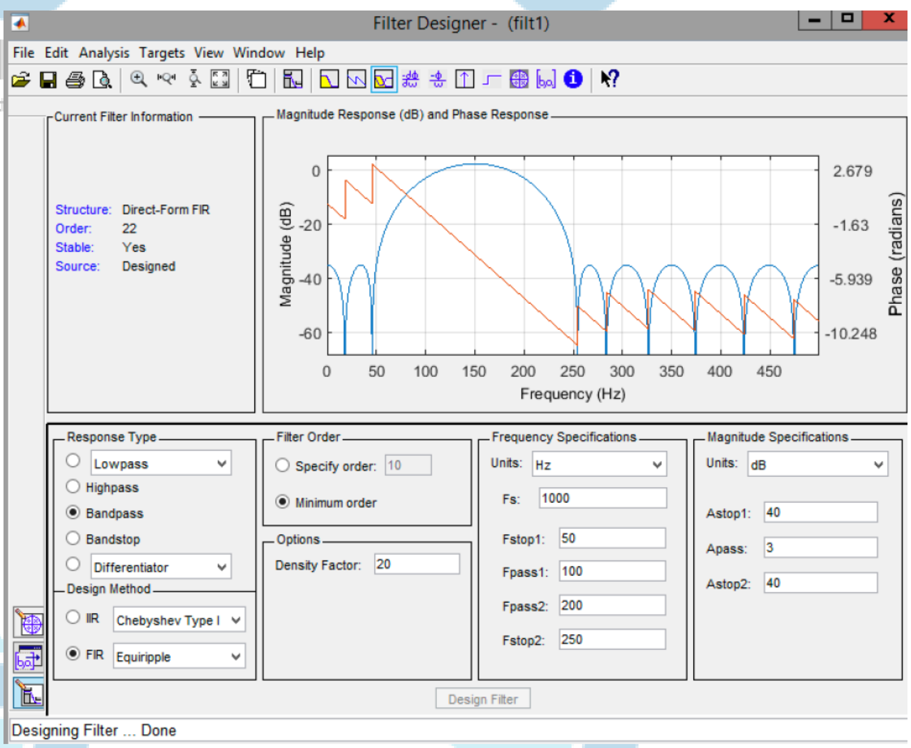


order of the filter

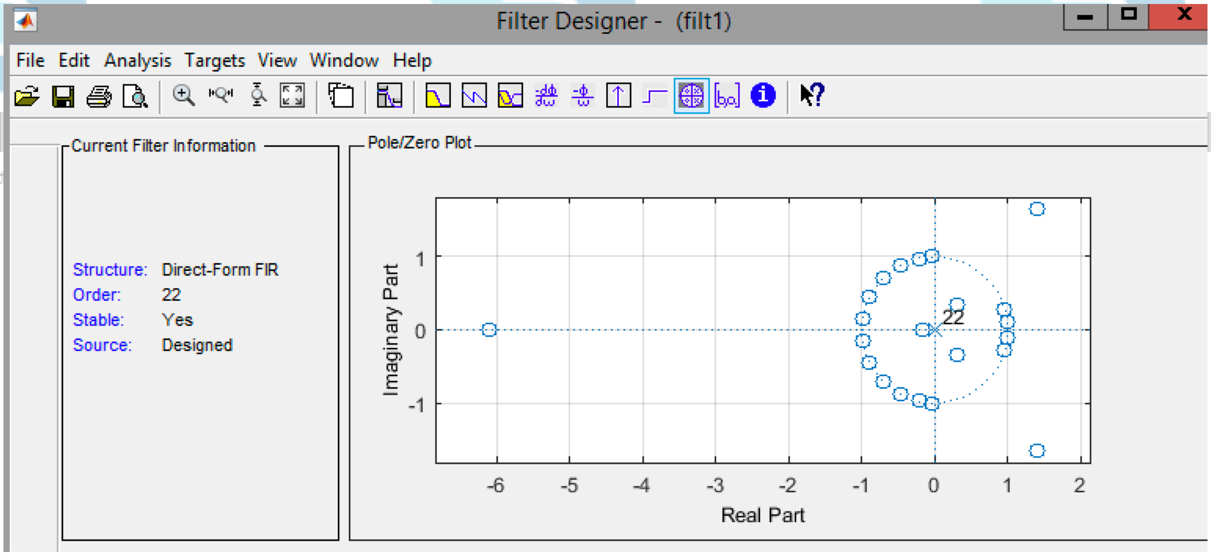
Ten is the order of the filter based on the number of poles, as the number of poles are not equal to the number of zero, so we will take the larger value as the order of a filter.

c) Design an equiripple FIR filter with the same specifications as in Task 2a.

Magnitude response and phase response of the filter



Pole/zero diagram of the filter



order of the filter:

22 is the order of the filter based on the number of zeros, as the number of poles are not equal to the number of zero, so we will take the larger value as the order of a filter which is number off zeros equal to 22.

d) Compare the magnitude response, phase response and order of three band-pass filters you have designed. This task can be completed when you write your report after the lab class.

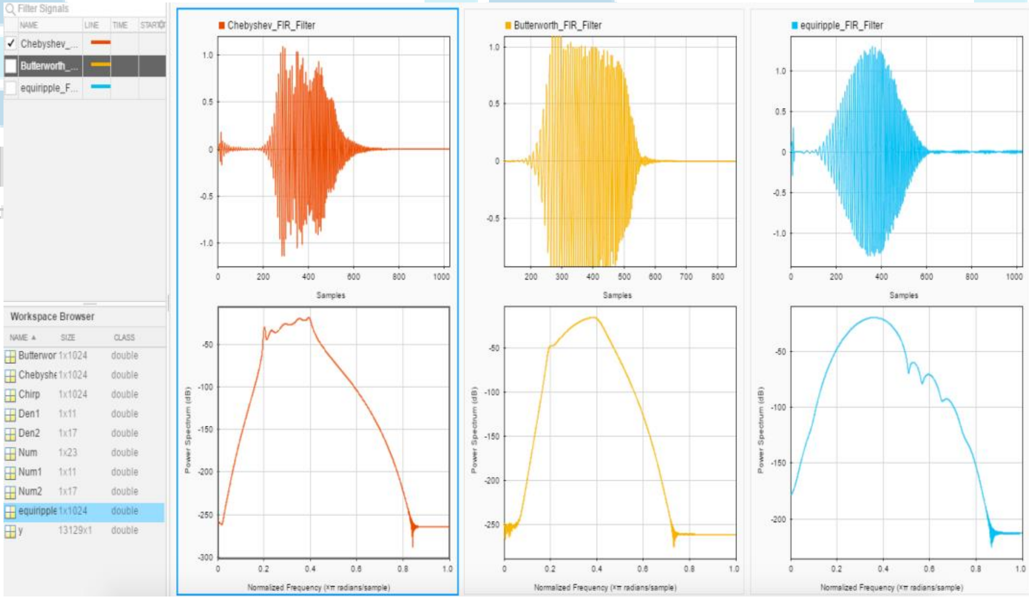
The requirements for this task (Task 2d) are listed below.
i. Describe the difference of three filters in terms of magnitude response;

ii. Describe the difference of three filters in terms of phase response; iii. Describe the difference of three filters in terms of filter order.

e) Filter the signal ‘Chirp.mat’ with each of the above band-pass filters. The sampling frequency of the ‘Chirp’ signal is 1000 Hz

The requirements for this task (Task 2e) are listed below.

i. Save the figures showing three output signals in the time domain and in the frequency domain and then include the figures in your lab report.



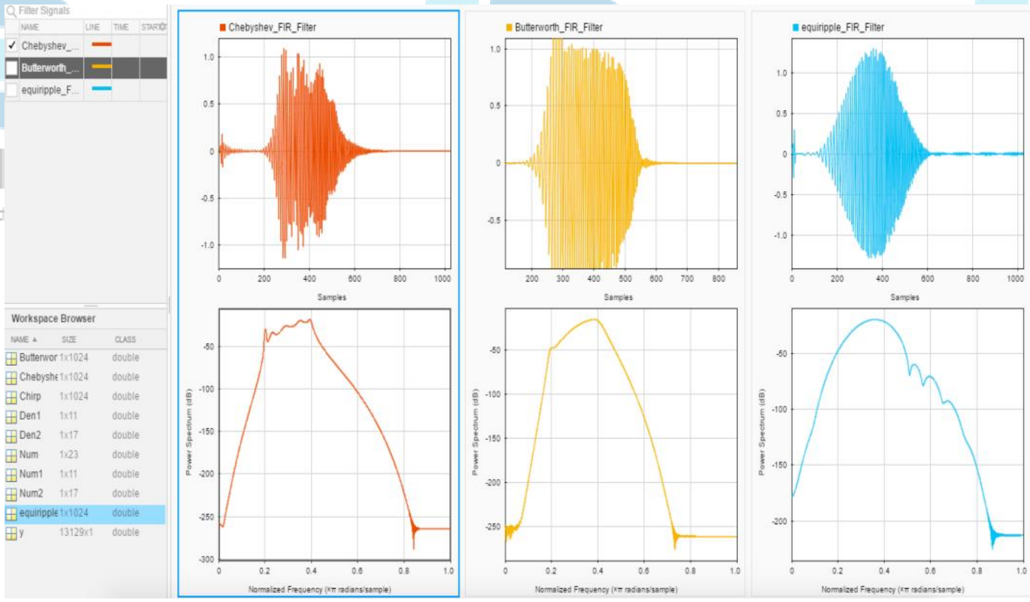
ii. Describe the difference between the input signal and the output signal in the time domain and in the frequency domain for each filter.

ANSWER: In the case off equiripple filter the plot is smoothly increasing and decreasing between the maximum y-axis value of 1.5 and -1.5 while in case of other two filters in time domain there are distortions in the plot.

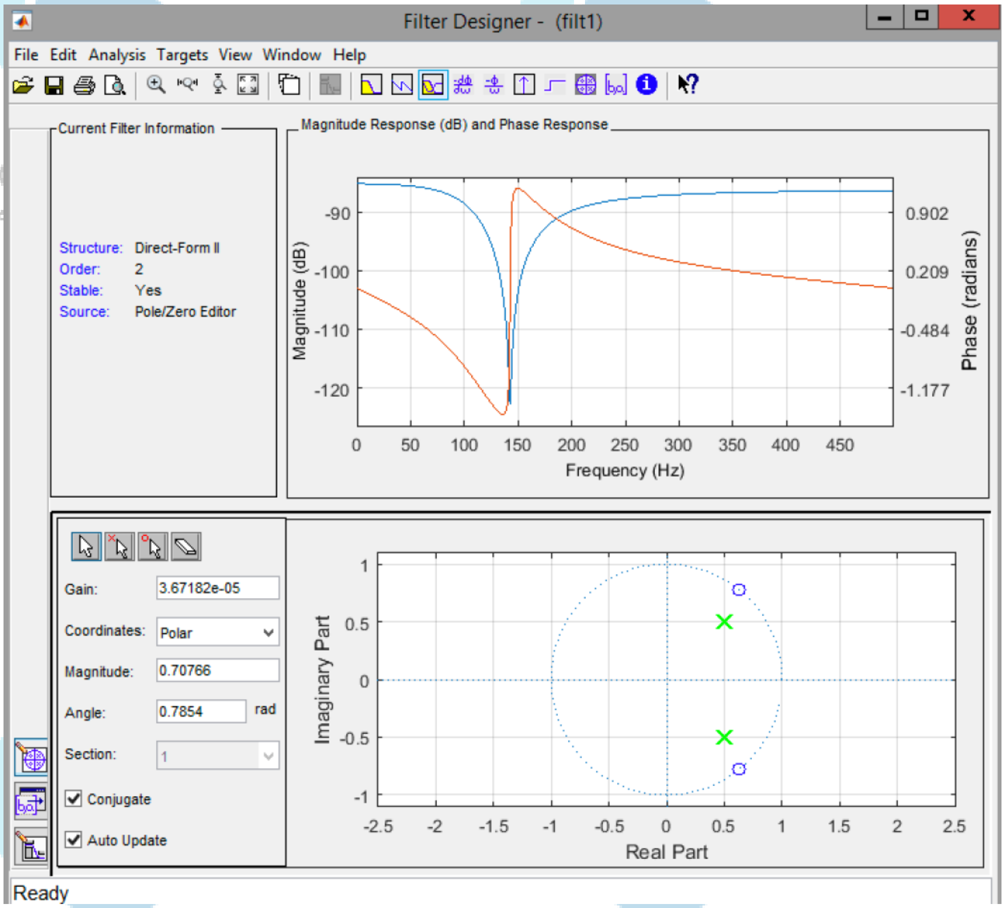
Where as in the frequency domain the chebyshev filter and equiripple filter stabilizes from the same frequency while in the Butterworth the filter starting stabilizing a little earlier form a frequency 0.75 Hz.

iii. Describe the difference of the filtering results using three filter

ANSWER:



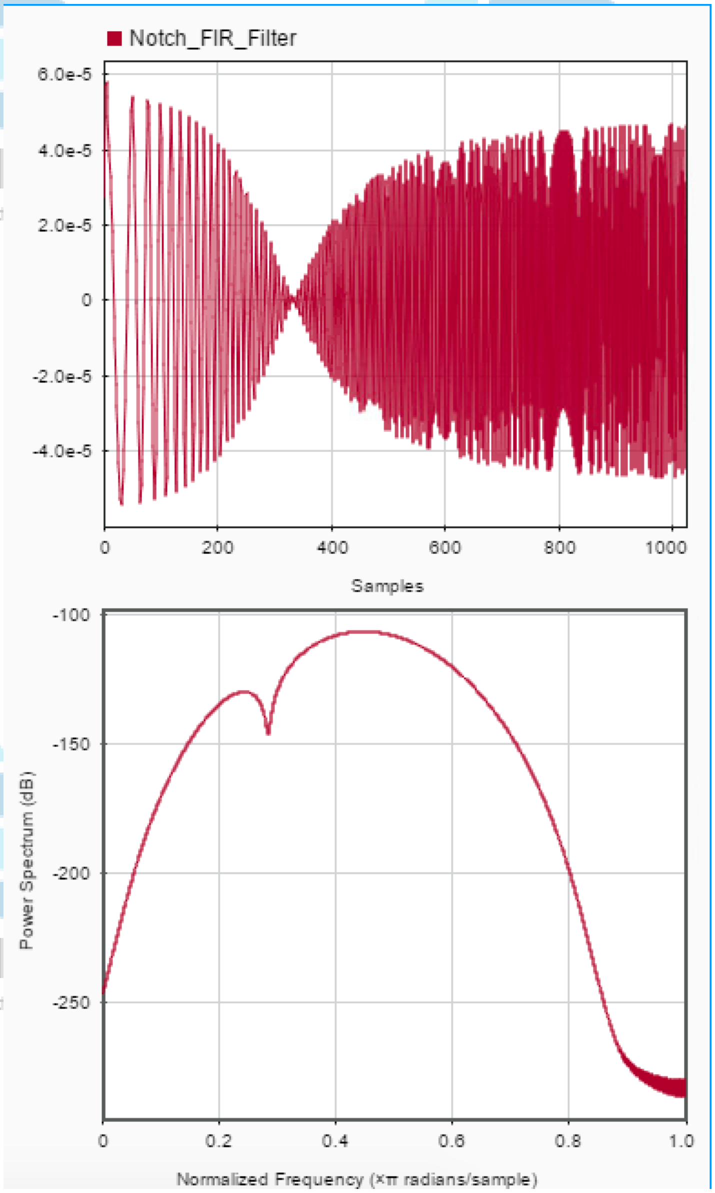
3a) Design a notch filter that has two poles and two zeros described in Section II Pre-work using the pole-zero placement method. Use the coordinates of the poles and zeros that you have found in your 'pre-work' for the design



3b) Filter the signal ‘Chirp.mat’ with the notch filter. The sampling frequency of the ‘Chirp’ signal is 1000 Hz.

The requirements for this task (Task 3b) are listed below.

i. Save the figures showing the output signal in the time domain and in the frequency domain and then include the figures in your lab report.



ii. Describe the difference between the input signal and the output signal in the time domain and in the frequency domain.

IV. LAB WORK PART B:

While performing the Pre-work part of this lab I faced many difficulties in the mathematical solution of the parts a to h.

1. What is the aim of the task? In other words, what do you want to do?

Answer: Mathematical work, calculation, designing and defining of filters IIR filter and FIR filter is the main of aim this task. We have defined both types of filter in the table-1 given below. Following the definition of two filters we are required to write the system and difference equation for both the filters. The next steps aim on sketching a signal using the MATLAB software by writing a suitable code for the requirements. Finding the poles and

zeros of a notch IIR filter with the specifications given and drawing a diagram for the result is also a requirement for this part.

2. Why do you want to do it?

Answer: In order to complete the table 1 attached in this document I have to do the mathematical pre-work, results and SS are given in question 5.

3. What are the challenges of this task when compared to the set tasks in Part A above?

Answer: For me the PART A was not as challenging as were the pre-work tasks are, because the Part A was quite simple we only have to design the two types of filter with a further sub division of three types, using the SP tool feature of MATLAB. This feature is quite easy and convenient to use, contrast to the mathematical computations done for getting the results of table 1 in pre-work.

4. How do you do it? In other words, can you explain your method or implementation?

Answer: By referring to the course book, I solved the questions, (a to h). Using the diagrams and pre-defined laws and definitions of the filter I implemented and solved the pre-work questions.

5. What are your results? Can you explain them?

Answer:

II. Pre-Work:

(a)

Answer:

IIR filter:

IIR filter requires the current output data and past output data as well. The performance of IIR filter is considered better, as it uses a feedback mechanism which results in a infinite impulse response.

FIR filter:

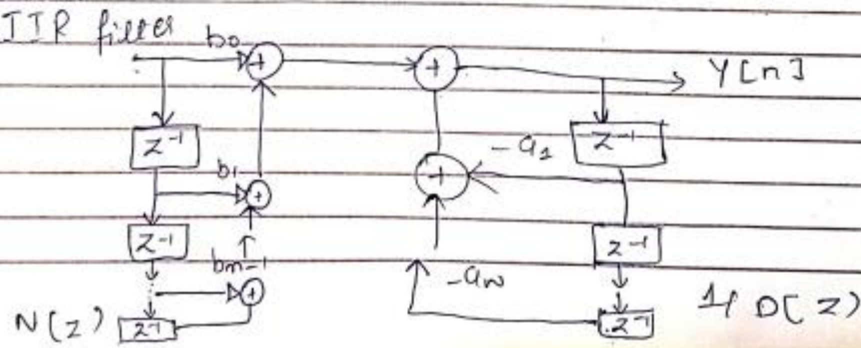
Comparing to the IIR filter the response of FIR filter is finite, as in a finite period of time it settles to zero.

The major use of FIR filter is when we need a linear phase and memory is provided by the system under observation.

(b)

Solution:

IIR filter



System Function Form

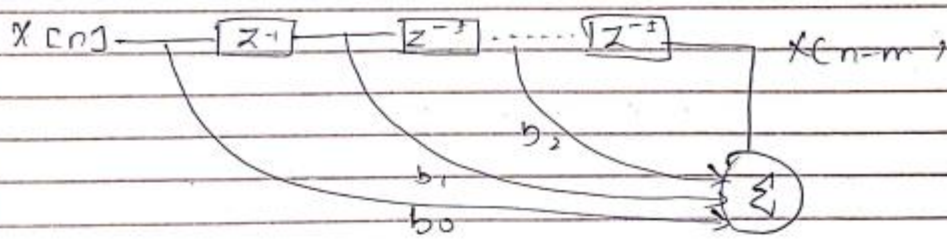
$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$$

In Difference equation form.

$$y(n) = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

CCI
Solution

FIR



System equation form.

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$= b_0 \prod_{k=1}^{M} (1 - c_k z^{-1})$$

Difference equation form.

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

(d)

Solution

Data:

pass-band edge frequency = 2000 Hz overall of the system.

F_s = sampling frequency = 10000 Hz

$$T = \frac{1}{10 \times 10^3}$$

T = 0.1 m sec

Analog frequency = $2\pi \times 2000$

$= 4\pi \times 10^3$ rad/sec

Discrete frequency (ω) = analog frequency $\times$ T

$= 4\pi \times 0.1 \times 10^{-3+3}$

$= 0.4\pi \times 10^0$

$= 0.4\pi \times 1 = 0.4\pi$ rad/sec.

(e) Calculation + Matlab

Calculation:

Data:

$F_s = 1000$ Hz

$f_p = [0, 200]$ Hz

$f_s = [250, f_s/2]$ Hz

passband ripple = 3dB
Stopband attenuation = 40dB.

Solution:

- $\omega_p = 2\pi \frac{f_p}{F_s} = 2\pi \times \frac{200}{1000} = 0.4\pi$ rad.

- $\omega_s = 2\pi \frac{f_s}{F_s} = 2\pi \times \frac{250}{1000} = 0.5\pi$ rad

$$-20\log_{10}|H(\omega)| = 40$$

$$\log_{10}|H(\omega)| = -\frac{40}{20} = -2$$

$$|H(\omega)| = \log_{10}^{-1}(-2)$$

$$|H(\omega)| = 10^{-2}$$

$$|H(\omega)| = 0.01$$

So, the response in the passband is within the interval.
$1 \pm \delta$

$$20\log_{10}(1+\delta) = 3$$

$$1+\delta = \log_{10}^{-1}\frac{3}{20}$$

$$1+\delta = 10^{0.15}$$

Therefore,

$$\delta = 1.412 - 1$$

$$\delta = 0.412$$

Then,

$$1 - 0.412 \leq |H(\omega)| < 1 + 0.412$$

So, we can write the interval as,

$$\boxed{0.588 \leq |H(\omega)| < 1.412}$$

(f)

Data: Given:

Sampling frequency $F_S = 1000$ Hz

Pass-band = [100, 200] Hz

Stop-band 1 = [0, 50] Hz

Stop-band 2 = [250, Fs/2] Hz

Pass-band ripple = 3dB

Stop-band attenuation = 40 dB

Signal $20\log_{10}|H(j\Omega)| = 20\log_{10}|H(j2\pi F)|$

Solution:

Frequency range [0, Fs/2] Hz

$$\omega = \frac{2\pi f}{F_S}$$

Pass Band Frequency $= \frac{2\pi(2000)}{1000} = 0.4\pi$ rad

Stop band frequency $= \frac{2\pi(250)}{1000}$

$= 0.5\pi$ rad.

So, it is within the interval $1 \pm \delta$

$20\log(1+\delta) = 3$

$\delta = e^{0.15} - 1 = 0.16183$

"So, the passband's frequency response is within the interval."

Result:

$0.8382 \leq H(\omega) < 1.6183$

★ Stop band maximum values $= |H(\omega)| < 10^{\frac{-40}{20}} = 0.01$

Calculation:

Given:

Data:

Sampling frequency $F_S = 1000\,Hz$
Notch frequency $F_O = 250\,Hz$
3 dB band-width of the notch filter $BW = 10\,Hz$

* By placing a pair of complex zeroes at points on the unit circle we can reject a 250 Hz component.

Now, $w_0 = \pm \frac{2\pi f_0}{f_s}$

for 250 Hz.

$$w_0 = \pm \frac{2\pi f_0}{f_s} = \pm \frac{2\pi \times 250}{1000} = 0.5\pi$$

converting in degrees $= \frac{\pi}{2} = 90^\circ$

Formula:

For calculating the pole magnitude:

$$\boxed{R = 1 - \frac{4w}{2}}$$

(iii) putting the values of

$R = 1 - 2\pi \frac{\Delta f}{f_s / 2}$

$\Rightarrow \boxed{0.938}$ → Answer for $\boxed{R}$

(b)

→ Diagram:

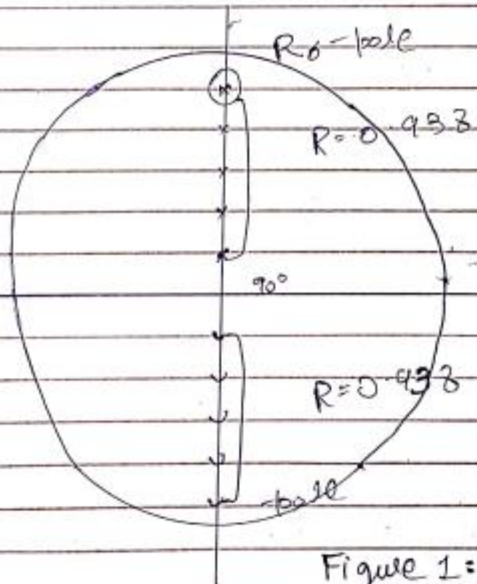


Figure 1:

The above ~~filter is~~ figure is filter pole zero

6. What can you conclude from the results?

Answer: If we use right techniques for solving a respective question we can obtain are desired results, for the example of this you can refer to the screenshots attached in question 5.

7. Have you achieved your aim?

Answer: **YES.**

8. Is your conclusion significant and why?

Answer: As, I apparently obtained the right answers of the questions, so, the conclusion was quite significant.

V. Conclusion:

In this experiment we have done the designing and the testing of different kinds of filter using MATLAB, signal analyzer and SP tool. Moreover, the Pre-work was mostly theoretical based, including calculations and sketching of some graphs, attached in the table 1. We have sketched the magnitude of the signals in decibels for continuous and discrete signals given in the pre-work, by finding the |H(w)| of the signal for the given time interval. This step was performed from part e and f also. In the g part we have calculated the value of the poles which is in this case was R = 0.938. By using the value of R we will plot filter pole zero as shown in the part h of the pre-work.

In the lab work using MATLAB and SP tool we designed the low pass and band-pass filter with a further sub division in three categories as given below:

- Butterworth IIR filter
- Chebyshev Type I IIR filter
- Equiripple FIR filter

The screenshots of magnitude response and phase response are attached above in the document. Using the signal analyzer, we have displayed the input and output signal in the frequency domain. As per the requirement of the question the zeros and poles figure of all the three types of filters is attached in the part A of the lab work. We have designed a notch filter by taking two zeros and two poles in the direction such that both the intersect at the origin.

In the part B we have to differentiate our work from others by selecting a type of problem we have faced during the practical performance of this lab.